How to Simulate a Trebuchet Part 4: Efficiency of the Trebuchet

There are many possibilities for calculating the "efficiency" of the trebuchet; three are presented here. All rely, at least initially, on determining the effectiveness of the trebuchet in converting its stored potential energy before launch into the kinetic energy of the flying projectile after launch.

The potential energy stored in the counterweight is consumed by the following actions:

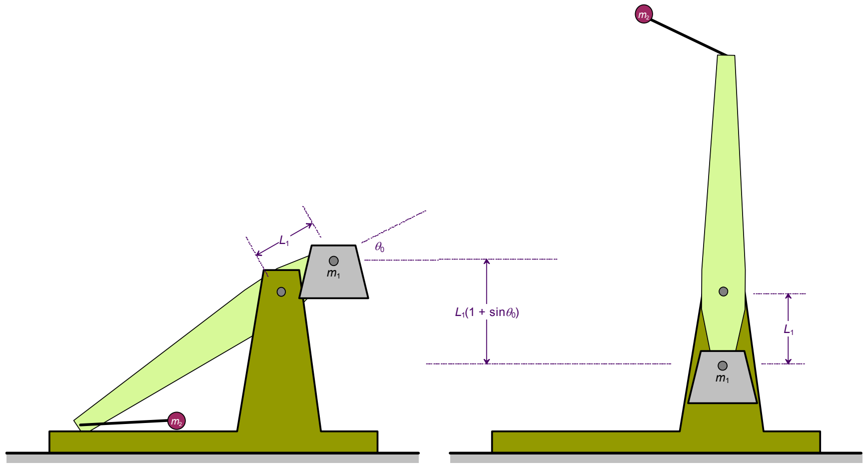
1. The kinetic energy of the projectile is increased from zero (before launch) to its maximum value at the moment of release from the sling.
2. The potential energy of the projectile is raised from zero (before launch) to a higher value depending upon the height at which the projectile is released.

Thus, the total potential energy that can be converted to kinetic energy of the projectile is

$$PE = PE_1 - PE_2 \qquad (1)$$

where PE_1 is the potential energy stored in the counterweight and PE_2 is the potential energy needed to raise the projectile to its launch height.

There are a number of other actions which tend to "steal" from the energy available to the projectile; for example the energy needed to accelerate the throwing arm (kinetic) and to raise the arm to its launch position (potential). It might seem appropriate to subtract these from PE in Equation (1), since the trebuchet cannot use this energy to launch the projectile. In fact, the energy needed to accelerate and raise the arm is a measure of the *inefficiency* of a trebuchet – a heavy arm with high moment of inertia is the mark of an inefficient device. We would thus prefer to penalize the efficiency of the trebuchet for the energy needed to move the arm. On the other hand, the energy needed to raise the projectile should not be held against the trebuchet since the designer has no control over the mass of the projectile chosen by the user.



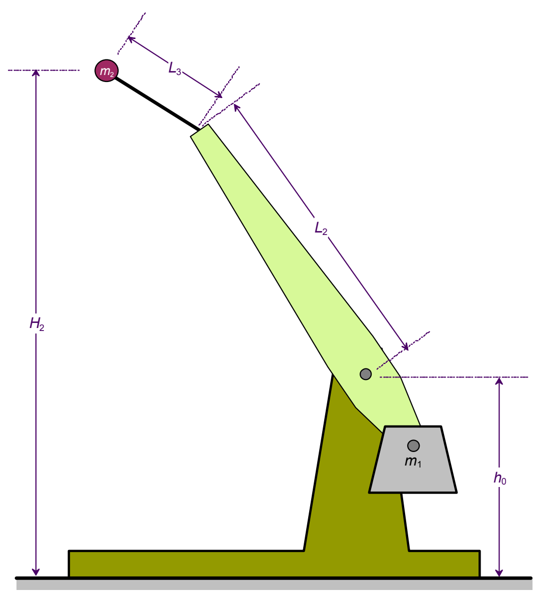
Potential Energy

The source of energy for the trebuchet is the change in height of the counterweight. As shown in the diagram above, the maximum possible vertical distance moved by the counterweight during a launch is

$$H_1 = L_1(1 + \sin\theta_0) \tag{2}$$

where θ_0 is the initial angle of the trebuchet arm. Thus, the potential energy stored in the counterweight is

$$PE_1 = m_1 g H_1 \tag{3}$$



The projectile must be raised to its launch height, which uses some of the potential energy of the counterweight. As shown in the diagram above, the launch height of the projectile is

$$H_2 = h_0 - L_2 \sin\theta_0 + L_3 \sin\psi_0 \tag{4}$$

where h_0 is the height of the arm pivot. The potential energy used to raise the projectile is then

$$PE_2 = m_2 g H_2 \tag{5}$$

and the total potential energy available for launching the projectile is

$$PE = m_1 g H_1 - m_2 g H_2 \tag{6}$$

Kinetic Energy

The kinetic energy of the projectile at launch is

$$KE = \frac{1}{2} m_2 v_0^2 \tag{7}$$

where v_0 is the velocity of the projectile at launch. The simplest way to measure the efficiency of the trebuchet is to divide the kinetic energy of the projectile [Equation (7)] by the available potential energy [Equation (6)]. We denote this the *energy efficiency*.

$$\eta_E = \frac{KE}{PE} \tag{8}$$

This is a very straightforward quantity to calculate after the trebuchet simulation has been run, and has the further advantage of being simple to understand. One disadvantage to the energy efficiency measure is that it takes no account of the direction in which the projectile is launched, or even whether it is launched forward at all! Clearly, the range of the projectile should enter into our efficiency calculations.

Range of the Projectile

If a projectile is launched from ground height, its maximum range is given by

$$R_0 = \frac{v_0^2}{g} \tag{9}$$

which assumes an optimum launch angle of 45°. If all of the available potential energy is converted to the kinetic energy of the projectile, the initial velocity (squared) of the projectile at launch is

$$v_0^2 = \frac{2PE}{m_2} \tag{10}$$

so that the range is

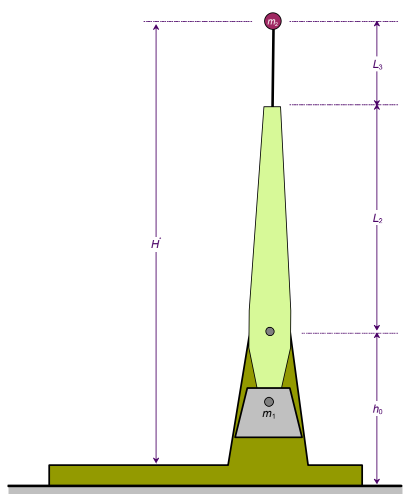
$$R_0^* = \frac{2PE}{m_2 g} \tag{11}$$

As the figures above show, however, the trebuchet releases the projectile at a significant distance above the ground. Because of this, the optimum range of the projectile is higher than the one given in Equation (9). The method for finding the new optimum range uses straightforward ballistic equations and differential calculus, but can become quite cumbersome if one does not employ the clever trick explained in Mungan [1]. The optimal range for a projectile launched above the ground is

$$R^* = R_0^* \sqrt{\frac{2H_2}{R_0^*} + 1} \tag{12}$$

which reduces to R_0^* if the launch height is zero, as expected. We can define a *range efficiency* by comparing the actual range to the range that would have been reached if all of the potential energy had been converted to kinetic energy and used to launch the projectile at the optimal angle.

$$\eta_R = \frac{R}{R^*} \tag{13}$$



Note, however, that the launch height might not be the best that is achievable with a given trebuchet. The *maximum* launch height is obtained when both the arm and sling are vertical, and is given by

$$H^* = h_0 + L_2 + L_3 \tag{14}$$

Thus, the absolute best range obtainable from a given trebuchet is

$$R_H^* = R_0^* \sqrt{\frac{2H^*}{R_0^*} + 1} \tag{15}$$

and a *height efficiency* can be defined as

$$\eta_H = \frac{R}{R_H^*} \tag{16}$$

We have defined three measures of the efficiency of a trebuchet. Of these, the energy efficiency will take on the highest value, but does not tell us whether the projectile has been launched effectively. A low value for energy efficiency can be the result of

- The arm is too heavy or has a high mass moment of inertia, compared with the mass of the counterweight and projectile.
- The proportions of arm and sling are incorrect, and do not allow the projectile to come up to its intended speed at launch.

On the other hand, a low range efficiency means that the projectile was launched in the wrong direction. This can usually be corrected by adjusting the hook angle, which controls the release time of the projectile.

Works Cited

[1] C. E. Mungan, "Maximum Range of a Projectile Launched from a Height," *The Physics Teacher,* p. 132, (41) 2003.